

Finite Math - Spring 2019

Lecture Notes - 2/28/2019

HOMework

- Section 3.3 - 27, 31, 33, 39
- Section 3.4 - 27, 30, 47, 49, 50

SECTION 3.3 - FUTURE VALUE OF AN ANNUITY; SINKING FUNDS

Annuities.

Definition 1 (Future Value of an Ordinary Annuity).

where

$$\begin{aligned}FV &= \text{future value} \\PMT &= \text{periodic payment} \\r &= \text{annual nominal interest rate} \\m &= \text{number of compounding periods per year} \\n &= \text{number of payments (periods)}\end{aligned}$$

Note that the payments are made at the end of each period.

Example 1. What is the value of an annuity at the end of 10 years if \$1,000 is deposited every 3 months into an account earning 8% compounded quarterly. How much of this value is interest?

Example 2. If \$1,000 is deposited at the end of each year for 5 years into an ordinary annuity earning 8.32% compounded annually, what will be the value of the annuity at the end of the 5 years?

Sinking Funds. We can turn the annuities picture around and ask how much we would need to deposit into an account each period in order to get the desired final value.

Example 3. New parents are trying to save for their child's college and want to save up \$80,000 in 17 years. They have found an account that will pay 8% interest compounded quarterly. How much will they have to deposit every quarter in order to have a value of \$80,000?

Example 4. A bond issue is approved for building a marina in a city. The city is required to make regular payments every 3 months into a sinking fund paying 5.4% compounded quarterly. At the end of 10 years, the bond obligation will be retired with a cost of \$5,000,000. How much will the city have to pay each quarter?

SECTION 3.4 - PRESENT VALUE OF AN ANNUITY; AMORTIZATION

Present Value of an Annuity. In the next concept, we will look at making a large deposit in order to have a fund which we can make constant withdraws from. We make an initial deposit, then make withdraws at the end of each interest period. We should have a balance of \$0 at the end of the predetermined amount of time the fund should last.

Example 5. *How much should you deposit into an account paying 6% compounded semiannually in order to be able to withdraw \$2000 every 6 months for 2 years? (At the end of the 2 years, there should be a balance of \$0 in the account.)*

Solution.

This gives rise to the following formula

Definition 2 (Present Value of an Ordinary Annuity).

where

$$\begin{aligned}
 PV &= \text{present value} \\
 PMT &= \text{periodic payment} \\
 m &= \text{number of compoundings per year} \\
 n &= \text{number of payments (periods)}
 \end{aligned}$$

Note that the payments are made at the end of each period.

Example 6. *How much should you deposit in an account paying 8% compounded quarterly in order to receive quarterly payments of \$1,000 for the next 4 years?*

Amortization. *Amortization* is the process of paying off a debt. The formula for present value of an annuity will allow us to model the process of paying off a loan or other debt. The reason the formula is the same is because receiving payments from your savings account is essentially the bank repaying you the money you loaned them by depositing it into a savings account.

Example 7. *Suppose you take out a 5-year, \$25,000 loan from your bank to purchase a new car. If your bank gives you 1.9% interest compounded monthly on the loan and you make equal monthly payments, how much will your monthly payment be?*

Example 8. *If you sell your car to someone for \$2,400 and agree to finance it at 1% per month (12% annual rate) on the unpaid balance, how much should you receive each month to amortize the loan in 24 months? How much interest will you receive?*

Solution.

Application. An interesting application of annuities uses a combination of present value and future value.

Example 9. *The full retirement age in the US is 67 for people born in 1960 or later. Suppose you start saving for retirement at 27 years old and you would like to save enough to withdraw \$40,000 per year for the next 20 years. If you find a retirement savings account (for example, a Roth IRA) which pays 4% interest compounded annually, how much will you have to deposit per year from age 27 until you retire in order to be able to make your desired withdraws?*

Example 10. *Lincoln Benefit Life offered an ordinary annuity earning 6.5% compounded annually. If \$2,000 is deposited annually for the first 25 years, how much can be withdrawn annually for the next 20 years?*

Solution.